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JET FORMATION IN GRAVITATIONAL FLOW OF A HEATED WAVY LIQUID FILM

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Jet formation was studied in the region of two-dimensional and three-dimensional waves in a heated liquid film flowing down a vertical surface. Jet-to-jet spacings were measured versus the film Reynolds number and the heat flow density. Three-dimensional waves on the film surface were formed naturally or by artificial perturbations. In addition to the thermocapillary mechanism of jet formation, a thermocapillary–wavy mechanism was found to exist.

Key words: *liquid film, jet formation, wavy flow, thermocapillary effects.*

INTRODUCTION

Studies of the instability of liquid film flows, the occurrence of three-dimensional deformations, and the development of jet flows on a film surface are important for understanding the mechanisms of heat transfer and crisis phenomena during heat transfer to the film.

Previous experimental studies have dealt primarily with nonisothermal flow of water films down vertical tubes 0.3–3.6 m long in the range of Reynolds numbers Re = 20–1000 [1, 2]. Film disruption occurs at the bottom of the heated part of the tube and rapidly spreads upward, leading to redistribution of the liquid flow rate over the tube perimeter [3] and deterioration of heat release. In the experiments of [1–3], formation of regular jets on the film surface was not observed because long vertical tubes (longer than 0.3 m) with diameters of less than 30 mm were used for large values of the film Reynolds number. As a result, considerable temperature gradients did not form on the liquid film surface; thermocapillary effects manifested themselves in the presence of developed three-dimensional waves and, without having a determining effect on the wavy pattern, led to film disruption.

The studies of [4, 5] revealed the formation of regular three-dimensional structures in a liquid film flowing down a plane with a small-size heater (6.5×13 mm). In the experiments, considerable temperature gradients (up to 15 K/mm) on the film surface were attained. The formation of structures during heating of a liquid film on various small-size heaters was studied in [6].

Isothermal flow of a film of a 31% glycerin solution in water down an inclined surface and development of three-dimensional waves were studied by Liu et al. [7]. Two-dimensional waves on the film surface were produced artificially by pressure perturbations with a particular frequency in the film former. In the upper part of the working segment, two-dimensional waves formed, which then disintegrated into three-dimensional synchronous waves and transformed to solitons. In [7], it is also shown that the wavelength of transverse three-dimensional perturbations decreases as the Reynolds number increases.

There have been extensive theoretical studies of the motion and stability of isothermal and heated liquid films. Many issues of the hydrodynamics of wavy flows of liquid films are analyzed in [8]. In film flows, the substantial Marangoni effect leads to the occurrence of several new types of instability. In uniformly heated falling liquid films at small Reynolds numbers, thermocapillary effects enhance wave instability. Their nonlinear mutual influence leads to disturbance of the two-dimensional wave pattern, occurrence of a wavy surface in the transverse direction, and jet formation. Further development of this instability can result in film disruption [9].

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b

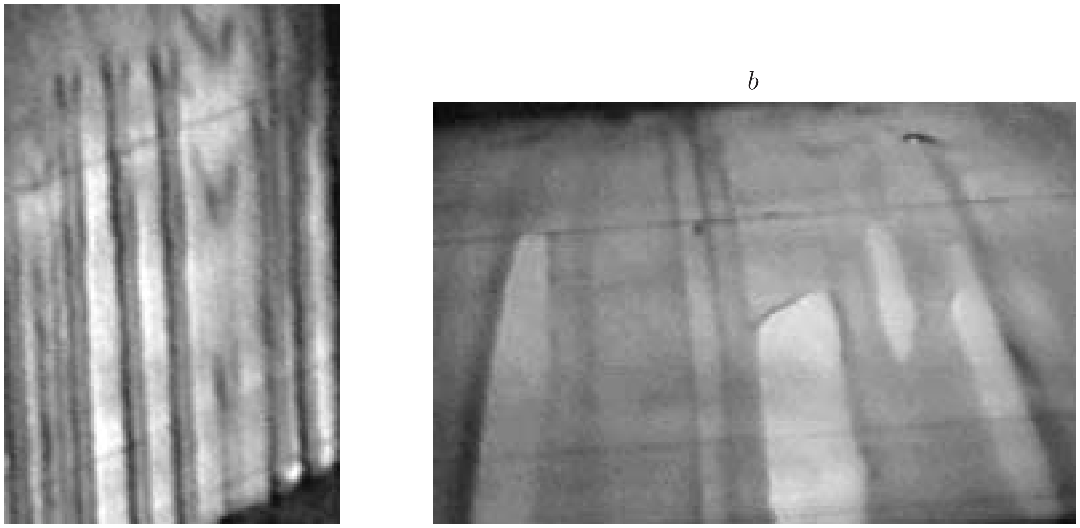

Fig. 1. Photographs of water film flow for Re = 10.4, X = 41.5 mm, and $\langle q \rangle$ = 0.64 (a) and 0.74 W/cm^2 (b).

At present, the mechanism of jet formation has not been completely clarified. There is no experimental information on the occurrence of jet flows during motion of liquid films on medium-size heaters under the joint influence of thermocapillary forces and wave formation at the interface.

In the present study, we experimentally investigated the formation of regular jets in liquid film flow in the region of two-dimensional and three-dimensional waves over a vertical heater with dimensions of 150 × 150 mm. Experimental data were obtained for water and FC-72 dielectric liquid. The film Reynolds number varied from 1 to 330. The initial temperature of the liquid film was 17–28°C.

1. EXPERIMENTAL FACILITY

The test facility was a closed circulation contour which incorporated a reservoir with a pump, a working area, a filter, rotameters, pipelines, and a stop valve. The working liquid was supplied by a pump to a film former which consisted of an accumulating chamber, a distribution device, and a nozzle with a calibrated flat slot. The liquid flowing down a plate was accumulated in a receiver and was then returned to the system under gravity. A detailed description of the experimental facility and the measuring procedure is given in [10].

A special feature the present studies was that for small Reynolds numbers and small values of the heat conductivity of FC-72 liquid, the local density of the heat flow differed considerably from the mean value. The local density of the heat flow q was determined from the temperature gradient on a furnace made of stainless steel, and the mean density of the heat flow $\langle q \rangle$, from the heater power. The film former was mounted at $X = 41.5$ and 120.0 mm from the upper edge of the heater.

2. RESULTS OF EXPERIMENTS

Water Film Flow. For Re < 6, an almost smooth film flowed down the heater. An increase in the heat flow density for Re = 1.1 did not give rise to considerable strains of the liquid film surface until film disruption. For Re = 2.4 and 4.9 and the values $\langle q \rangle$ = 0.3–0.4 W/cm^2 preceding film disruption, weak heterogeneities in the form of vertical jets were observed on the surface.

Figure 1 shows photographs of water film flow for Re = 10.4 and X = 41.5 mm. For $\langle q \rangle > 0.3$ W/cm^2, one can see vertical heterogeneities on the liquid film surface are evident. At the top, the jets are discontinuous but their surface is affected by three-dimensional waves; at the bottom, the jet shape is more even and raised and the

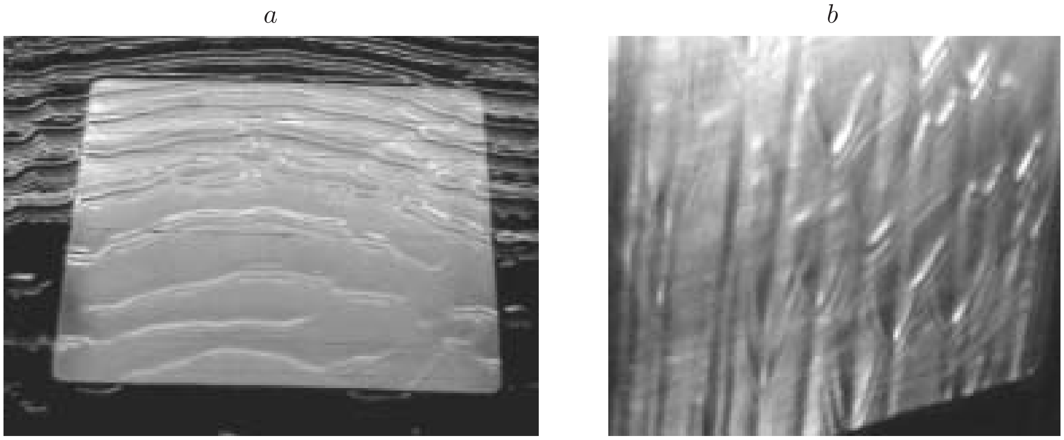

Fig. 2. Photographs of water film flow for Re = 22, $X = 120$ mm, and $\langle q \rangle = 0$ (a) and 0.91 W/cm^2 (b).

film between the jets becomes even and more smooth (Fig. 1a), as in the case of smaller Reynolds numbers. Jet formation occurs at the end of the smooth zone at 70–80 mm from the film former nozzle, where wave motion occurs on the film surface. As the heat flow density increases, the mean jet-to-jet spacing decreases from 20 to 15 mm. Dry spots (part of the heater surface not covered by the liquid) appear, as a rule, at the bottom of the heater and propagate upward between the jets (Fig. 1b).

For Re = 44.2, three-dimensional waves on the film surface were observed at 30–40 mm from the upper edge of the heater. As the heat flow density increased, jets formed but they were less pronounced and consisted of alternating crests and troughs of three-dimensional waves.

Change in the Wavy Flow on the Film Surface. The distance between the nozzle and the heater was increased to 120 mm to change the wavy pattern of the film flow. Photographs of the film flow for Re = 22 are shown in Fig. 2. It is evident that two-dimensional waves flow onto the top of the heater. In the case of no heat flow and small values of $\langle q \rangle$, two-dimensional waves are deformed when passing over the heater but their crestedness is weakly expressed (Fig. 2a). For large heat flow densities at the bottom of the heater, three-dimensional waves develop on the film surface. Furthermore, as for $X = 41.5$ mm, an increase in the heat flow density leads to a decrease in the distance between the three-dimensional wave crests. Liquid jets form (Fig. 2b). A similar pattern is observed at Re = 33. In this case, three-dimensional waves also form at the top of the heater.

Artificial Breakup of Two-Dimensional Waves into Three-Dimensional. Artificial formation of three-dimensional waves was performed at 17 mm from the upper edge of the heater using cylinders of 2 mm diameter placed in a liquid film. The distance l_w between the cylinders was varied from 5 to 40 mm with a step of 5 mm. In the absence of a heat flow, no changes in the three-dimensional wave pattern were observed for various values of l_w. Photographs of water film flow for $\langle q \rangle = 0.91$ W/cm^2 are presented in Fig. 3. A comparison of the photographs in Fig. 2b and Fig. 3 shows that the wavy structure of the film and the number of jets are significantly affected by the distance l_w. The number of jets (in our case 13) coincided with the number of cylinders only for $l_w = 10$ mm. For $l_w = 20$ mm, 12 jets were observed. For $l_w = 5$ mm, the number of jets decreased to 10, and for $l_w = 15$, 30, and 40 mm, eight or nine jets formed, as in the case of an unperturbed film. Thus, there is a perturbed region ($l_w = 5$, 10, and 20 mm) in which the jet-to-jet spacing Λ decreased by 30–40%.

FC-72 Liquid Film Flow. In the FC-72 liquid film flow with Re > 5 over a 150 × 150 mm heater in a pressure-tight container, two-dimensional and three-dimensional waves are observed on the film surface. For Re = 5, weakly expressed transverse perturbations arise on the film surface near the upper edge of the heater even at low heat flow densities. In the remaining part of the heater, three-dimensional waves propagate on the film surface. For $\langle q \rangle = 0.17$ W/cm^2 jets are observed visually over the entire heater. Near the upper edge of the heater, the number of jets is larger ($\Lambda = 3$ mm). In the middle and at the bottom of the heater, 23 jets form; the mean spacing between them is 6.5 mm.

Variation in the jet-to-jet spacing is apparently caused by a nonuniform distribution of the local heat flow density along the heater [11]. The heat flow density along the heat source decreases monotonically because of heat transfer from the lower (hotter) part of the heater to the upper part. The heat transfer is determined by two

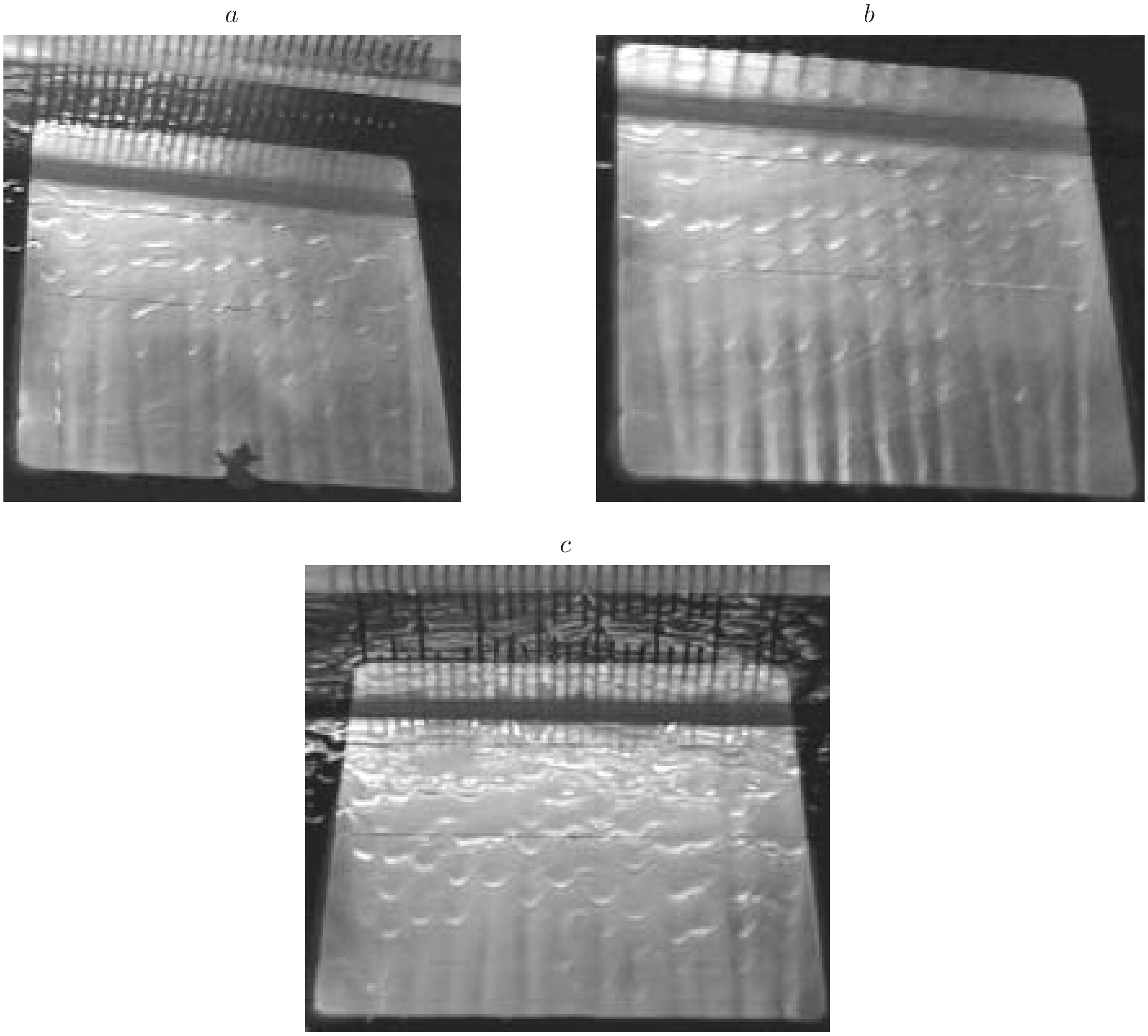

Fig. 3. Photographs of water film flow for artificial formation of three-dimensional waves (Re = 22, $X = 120$ mm, and $\langle q \rangle = 0.91$ W/cm^2) for $l_w = 5$ (a), 10 (b), and 20 mm (c).

independent factors: variation in the liquid bulk temperature and variation in the local heat release coefficient. The specific heat and thermal conductivity of FC-72 liquid are four and ten times smaller, respectively, than those of water. The heat flow density redistribution for FC-72 liquid is more significant, which is confirmed by measurements and calculations similar to those in [11]. With increase in Re on the heater surface, the distribution of the local heat flow density becomes more uniform.

For $\langle q \rangle = 0.2$ W/cm^2, the jet-to-jet spacing increases to 3.06 mm at the upper edge of the heater and decreases to 5.8 mm at the bottom. For $\langle q \rangle = 0.22$ W/cm^2, the motion of jets becomes unstable near the upper edge of the heater. For $\langle q \rangle = 0.25$ W/cm^2 a flow with even jets arises (a jet-to-jet spacing of 3.6 mm). For $\langle q \rangle = 0.32$ W/cm^2, this flow occupies more than half the heater surface area (Fig. 4). Dry strips are located between the jets. With a further increase in the heat flow density, the jet-to-jet spacing increases. For $\langle q \rangle = 0.35$ W/cm^2, the jets become unstable at the bottom of the heater and their oscillations are observed. For $\langle q \rangle = 0.43$ W/cm^2, 35 jets flow down over the entire length of the heater and the jet-to-jet spacing is 4.3 mm. The oscillation of the jets is enhanced, and at the bottom of the heater, they are evaporated. It should be noted that the evaporation of the liquid results in a decrease in the film Reynolds number and can be one of the reasons for variation in Λ.

For Re $\leqslant 10$, the waves at the top of the heater are nearly two-dimensional. Three-dimensional waves in the middle of the heater are visible more clearly. At both the top and bottom of the heater, jets on the film surface form simultaneously with the occurrence of film disruption. For Re = 14.4 and 20.3, jets form before film disruption, and

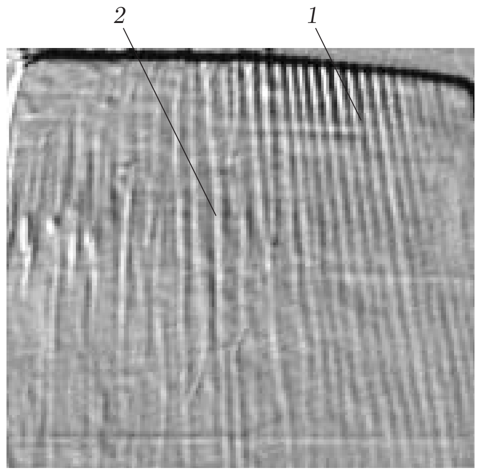

Fig. 4. Photograph of FC-72 liquid film flow for Re = 5 and $\langle q \rangle = 0.32$ W/cm^2: 1) type A jet flow; 2) type B jet flow.

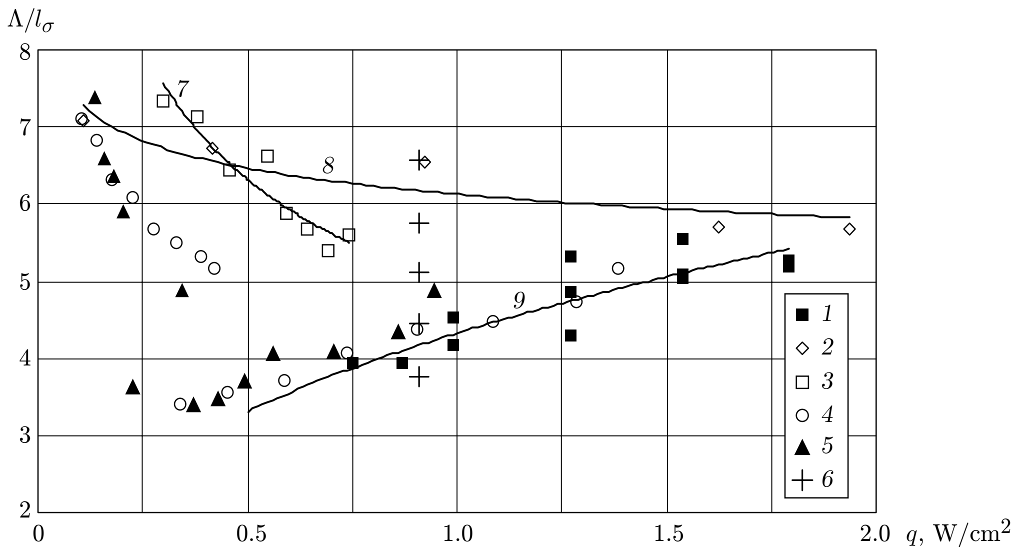

Fig. 5. Mean jet-to-jet spacing versus local heat flow density: points 1 refer to the MD-3F liquid (Re = 2, 6.5×13 mm heater) [5], points 2 and 3 refer to water for Re = 44.2 and 10.4, respectively, points 4 and 5 refer to FC-72 liquid for Re = 6.8 and 5, respectively, and points 6 refer to water for Re = 22 (flow with artificial perturbations); curve 7 is an approximation of points 3, curve 8 is an approximation of points 2, and curve 9 is an approximation of points 1.

at the top of the heater, they are poorly discernible. For Re = 51 and 73 in the case of a low-density heat flow or the absence of a heat flow, synchronous three-dimensional waves with a wavelength in the transverse direction of 6.0–6.8 mm are observed. With increase in $\langle q \rangle$, jets form from the synchronous three-dimensional waves, as is the case in water film flow. For Re > 100, the thermal layer enters the film surface at a considerable distance from the upper edge of the heater. The jets become less regular, and their number varies from 16 to 24.

Thus, it has been established that on medium-size heaters, the value of Λ can increase along the heater length by more than a factor of two. For convenience, jet flow with a smaller value of Λ at the top of the heater is called type A flow and jet flow in the middle and at the bottom is called type B flow. The division of the jet flows into two types is due to the fact that with increase in the heat flow density, the jet-to-jet spacing increases for type A jet flows and decreases for type B jet flows. For water film flow on the given heater, type A jet flow was not recorded; i.e., the formation of flows of different types depends on the type of liquid and boundary conditions on the substrate and the film surface.

3. ANALYSIS AND GENERALIZATION OF EXPERIMENTAL DATA

Figure 5 shows the dimensionless jet-to-jet spacing Λ/l_σ versus the local heat flow density (l_σ is a capillary constant). The data obtained for flow of a perfluorotriethylamine (MD-3F) film on a 6.5×13 mm heater [5]. In this case, the jet-to-jet spacing increases with increase in the value of q. A similar dependence for lower heat flow densities is observed for FC-72 liquid ($\mathrm{Re} = 5.0$ and 6.8) in the case of formation of type A structures.

In experiments with water (until the of formation dry spots), the values of Λ decrease as q increases. It should be noted that there is a considerable difference between the data of the present study for water at $\mathrm{Re} = 10.4$ and 44.2. For $\mathrm{Re} = 10.4$, the local heat flow density has a significant effect on the value of Λ, whereas for $\mathrm{Re} = 44.2$, the effect of q is much weaker. For $\mathrm{Re} = 10.4$, the jet formation mechanism is largely determined by thermocapillary forces because nonuniformity in the film thickness in the transverse direction arises when development of instability of two-dimensional waves is at an early stage. For $\mathrm{Re} = 44.2$, the effect of thermocapillary forces is much weaker, because it is superimposed on the flow with already developed three-dimensional waves.

For FC-72 liquid flow in the middle and at the bottom of the heater, as well as for water, the jet-to-jet spacing decreases with increase in q. From Fig. 5 it follows that as in experiments with water, the effect of the heat flow density on the value of Λ is enhanced as Re increases. Thus, it can be concluded that for perturbations that arise from breakup of three-dimensional waves in the case of formation of type B jet flow, the presence of a thermocapillary force whose magnitude is proportional to q leads to a decrease in the perturbation wavelength in the direction perpendicular to the film flow. Artificial perturbations on the film surface allow Λ to be decreased from the values corresponding to type B flows to the values corresponding to type A flows. It can be assumed that the limiting perturbation values that can be attained in heated liquid films correspond to these regimes.

Figure 6 gives the dimensionless jet-to-jet spacing versus Reynolds number. The values of l_σ were calculated with allowance for the slope of the plate. Figure 6a gives the mean jet-to-jet spacing versus Reynolds number for water. It is obvious that in the absence of a heat flow, the distance between the crests of three-dimensional wave does not depend on Re. For a nonisothermal film, the wavelength decreased with increase in the Reynolds number. In the dependence $\Lambda \sim \mathrm{Re}^n$, the exponent decreased from -0.028 to -0.310 as the heat flow density increased. Figure 6a also gives the data of [7] on the wavelength of synchronous three-dimensional waves for flow of an isothermal liquid film over a plate inclined at an angle of 6.4° to the horizon. The data of [7] are well below those obtained in the present study, and the wavelength of synchronous three-dimensional waves decreases with increase in Re. In our experiments, a similar influence of the Reynolds number was observed only for high heat flow densities. This difference can be explained by changes in the slope of the surface and in the dimensions of the working area. Another reason is that in the experiments of [7], the waves on the film surface were generated artificially using pressure pulses in the film former system.

Figure 6b generalizes data on the jet-to-jet spacing and compares them with previous results. In processing the data corresponding to small values of $\langle q \rangle$, we used the distance between the crests of three-dimensional waves instead of the jet-to-jet spacing. Figure 6b shows data obtained for film flows on a vertical surface with a 6.5×13 mm heater for perfluorotriethylamine [5] and a 25% solution of alcohol in water [12] and data for a sloping surface with 2.2×68 and 4×68 mm heaters [6] with formation of regular structures. The data obtained on a 150×150 mm heater for water are above the data obtained on heaters of small dimensions. For small heaters, the jet-to-jet spacing was always larger the higher the value of Re.

Figure 6b also gives results from two series of measurements of the dependence of Λ on Re for FC-72 liquid flow on a heater with dimensions of 150×150 mm. In the case of formation of type A jet flow ($\langle q \rangle = 0.4$ W/cm^2), the value of Λ agrees with the data obtained on small heaters. In the case of formation of type B jet flow ($\langle q \rangle \leqslant 0.2$ W/cm^2), Λ practically does not depend on Re, which agrees with the data obtained for water flow on a 150×150 mm heater. We note that the spread in data in experiments with FC-72 liquid is much larger than that in experiments with water. This is apparently explained by the absence of a distinct boundary between type A and type B jet flows.

Two mechanisms of jet formation in a nonisothermal falling liquid film can be distinguished. In the region of smooth flow of a liquid film on heaters of small dimensions, perturbations in the film are initially absent. Regular structures, revealed and explored in [4–6, 10, 11] (10 in Fig. 6b) form when the heat flow density reaches a threshold value. In liquid film flow on long heaters, jets develop in the region of three-dimensional instability of waves and jet flow forms gradually as the heat flow density and the distance from the upper edge of the heater increase. Data on the distances between the crests of three-dimensional waves at $\langle q \rangle = 0$ are shown by points 1 in Fig. 6a.

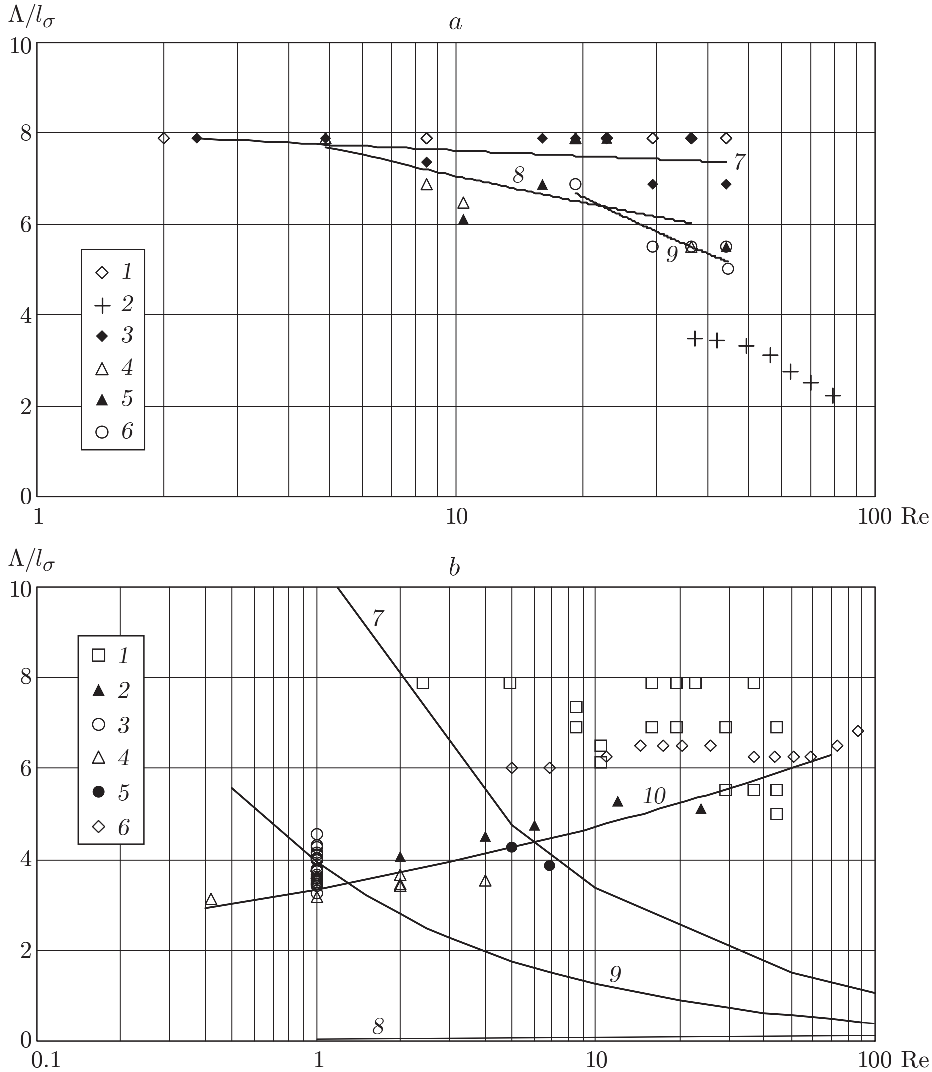

Fig. 6. Dimensionless jet-to-jet spacing versus Reynolds number: (a) points 1 and 3–6 refer to water for $\langle q \rangle = 0$, 0.3, 0.6, 0.9, and 1.2 W/cm^2, respectively; points 2 refer to the experimental data of [7]; curves 7 ($\Lambda/l_\sigma = 8.0\,\mathrm{Re}^{-0.023}$), 8 ($\Lambda/l_\sigma = 9.4\,\mathrm{Re}^{-0.12}$), and 9 ($\Lambda/l_\sigma = 16.6\,\mathrm{Re}^{-0.31}$) approximate points 3, 4, and 6, respectively; (b) points 1 refer to water for $\langle q \rangle = 0$–1.2 W/cm^2; points 2 refer to the MD-3F liquid (a heater with dimensions of 6.5×13 mm) [5]; points 3 to MD-3F liquid (heaters with dimensions of 2.2×68 and 4×68 mm) [6]; points 4 to a 25% solution of alcohol (a heater with dimensions of 6.5×13 mm) [12]; points 5 to FC-72 liquid (type A flow); points 6 to FC-72 liquid (type B flow); curves 7, 8, and 9 refer to the calculation of [8], [13], and [14], respectively; curve 10 ($\Lambda/l_\sigma = 3.34\,\mathrm{Re}^{0.15}$) refers to regular structures [5].

Comparison with theoretical data on the instability of three-dimensional waves can be only qualitative. Analyzing the Nepomnyashchii equation [15] for a vertical liquid film, Tsvelodub and Kotechenko [13] showed that instability develops for all perturbations with wavelengths greater than the value corresponding to curve 8 in Fig. 6b. Alekseenko et al. [8] analyzed the dispersion relation of an equation similar to the Nepomnyashchii equation. For approximately identical perturbations in the transverse and longitudinal directions, the neutral curve equation leads to the relation

$$\Lambda/l_\sigma = 2\pi 3^{0.5}\,\mathrm{Re}^{-0.5},$$

which corresponds to curve 7 in Fig. 6b. A similar dependence with a different coefficient (curve 9 in Fig. 6b) was obtained by Medvedkov and Sharypov [14], who studied the stability of two-dimensional liquid film flow with a nonuniform temperature distribution on the free boundary and derived conditions for transition to three-dimensional jet flow. Joo and Davis point out [16] that two-dimensional waves are always unstable and three-dimensional perturbations develop on a vertical surface. The critical wavelength of three-dimensional perturbations decreases with increase in Re, as is the case in [8, 14].

Thus, the experimental results obtained have not been explained theoretically. The absence of agreement with known theoretical results on the stability of an isothermal liquid film to three-dimensional perturbation suggests that it is necessary to allow for the mutual effect of wavy and thermocapillary effects.

CONCLUSIONS

The hydrodynamics of a nonisothermal wavy falling liquid film was studied experimentally. It the thermocapillary–wavy mechanism, vertical jets form on film heterogeneities at the crests of two-dimensional waves breaking up into three-dimensional waves or on synchronous developed three-dimensional waves. As the heat flow density increases, the liquid film between the jets becomes smooth and waves move over the surface of the jets. By means of artificial perturbations on the heated liquid film surface, the instability wavelength can be changed only in a narrow range that corresponds to the region in which the regularities of the thermocapillary and thermocapillary–wavy mechanisms of jet formation are manifested.

In FC-72 dielectric liquid flow, jet flows of two types were found, which differed in the nature of the dependence of the jet-to-jet spacing on the heat flow density. For a falling water film, the experiments revealed only one type of flow, for which the wavelength decreased with increase in the heat flow density.

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